

BRST - BFV analysis of anomalies in bosonic string theory interacting with background gravitational field

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Abstract

The general BRST-BFV analysis of anomaly in the string theory coupled to background fields is carried out. An exact equation for c-valued symbol of anomaly operator is found and structure of its solutions is studied.

1. All conventional string models are anomalous at the quantum level and only under certain conditions on parameters of a theory anomalies can be cancelled. In theories of a string interacting with background fields the role of parameters is playing by these fields and conditions of anomaly cancellation are interpreted as effective equations of motion for them [1-4]. Standard covariant method for deriving equations of motion for background fields in string theories is based on the principle of quantum Weyl invariance which demands that renormalized operator of the energy-momentum tensor trace vanishes. General structure of such an operator in the theory of a string coupled to massless background fields was studied in details in refs. [5,6] (see also the reviews [7,8]).

More consistent (from the theoretical point of view) method of investigation of anomalous theories is formulated within the framework of BRST - BFV quantization [9-12] (see also the book [17]). The BRST - BFV quantization method, or the method of generalized canonical quantization provides a powerful and universal approach to formulation of gauge theories. Condition of anomaly cancellation in this approach consists in preserving of gauge algebra of the theory at the quantum level. In the case of string theories it reduces to the nilpotency condition for the fermionic operator Ω generating quantum gauge algebra.

The explicit form of quantum operators Ω and Ω^2 in free string theory was constructed and critical values of parameters (such as space-time dimensions) were obtained from the nilpotency condition in refs.[13,14](see also [15,16]). Generalization of this approach to string models with background fields faces problems connected with very complicated structure of these theories. A string interacting with background fields is described by a σ -model type action that is essentially non-linear and so requires a suitable perturbation scheme. An attempt to construct such a scheme was undertaken in ref. [18] where the nilpotency condition was studied for bosonic string coupled to massless background fields within the expansion both in powers of \hbar and in normal coordinates. The lowest order was considered in details and correspondence with covariant approach was also discussed.

Some general features of the anomaly can be found using only general properties of the operator Ω without explicit computation of Ω^2 and so without construction of any perturbation scheme. In refs. [19,20] general structure of quantum anomaly in theories of free bosonic and fermionic strings was derived from the Jacobi (super)identity for the operator Ω and dimensional considerations. In this paper we consider on the same base the theory of bosonic string coupled to background gravitational field. Being a theory with non-polynomial interaction, this model provides a number of new aspects and details, but as we show the structure of anomaly can be determined up to a function of string coordinates and two constants.

2. The closed bosonic string interacting with background gravitational field is described by the action

$$S = -\frac{1}{2} \int d^2\sigma \sqrt{-g} g^{ab} \partial_a x^\mu \partial_b x^\nu G_{\mu\nu}(x). \quad (1)$$

Here $G_{\mu\nu}(x)$ is the metric of d -dimensional space-time with coordinates x^μ ; $\mu, \nu = 0, 1, \dots, d-1$. g_{ab} is the metric of the two-dimensional world sheet of the string, $\sigma^a = (\tau, \sigma)$ are coordinates on the world sheet; $a, b = 0, 1$.

The theory possesses two first class constraints [18]

$$\begin{aligned} L(\sigma) &= \frac{1}{4} G_{\mu\nu} (p_\mu - G_{\mu\alpha} x'^\alpha) (p_\nu - G_{\nu\beta} x'^\beta), \\ \bar{L}(\sigma) &= \frac{1}{4} G_{\mu\nu} (p_\mu + G_{\mu\alpha} x'^\alpha) (p_\nu + G_{\nu\beta} x'^\beta), \end{aligned} \quad (2)$$

satisfying Virasoro algebra (in terms of Poisson brackets)

$$\begin{aligned} \{L(\sigma) L(\sigma')\} &= -(L(\sigma) + L(\sigma')) \delta'(\sigma - \sigma'), \\ \{\bar{L}(\sigma) \bar{L}(\sigma')\} &= (\bar{L}(\sigma) + \bar{L}(\sigma')) \delta'(\sigma - \sigma'), \\ \{L(\sigma) \bar{L}(\sigma')\} &= 0. \end{aligned} \quad (3)$$

$p_\mu(\sigma)$ are momenta conjugated to $x^\mu(\sigma)$ and $x'^\mu = \partial_\sigma x^\mu$.

Let us introduce the canonical BRST charge

$$\Omega = \int_0^{2\pi} d\sigma \left(L(\sigma) \eta(\sigma) + \bar{L}(\sigma') \bar{\eta}(\sigma') - \mathcal{P}(\sigma) \eta(\sigma) \eta'(\sigma) + \bar{\mathcal{P}}(\sigma') \bar{\eta}(\sigma') \bar{\eta}'(\sigma') \right). \quad (4)$$

Here η, \mathcal{P} and $\bar{\eta}, \bar{\mathcal{P}}$ are ghost fields corresponding to constraints L and \bar{L} respectively. The charge Ω is a generator of BRST transformation $\delta B = \{B, \Omega\}_\epsilon$ where ϵ is a fermionic parameter and B is an arbitrary functional on extended phase space. In the case of the theory under consideration it leads to the following transformation properties for the fundamental variables:

$$\begin{aligned} \delta\eta &= -\eta \eta' \epsilon, \\ \delta\bar{\eta} &= \bar{\eta} \bar{\eta}' \epsilon, \\ \delta x^\mu &= \frac{1}{2} (\eta Y^\mu + \bar{\eta} \bar{Y}^\mu) \epsilon, \\ \delta p_\mu &= \frac{1}{2} \partial_\sigma (\bar{\eta} \bar{Y}_\mu - \eta Y_\mu) - \\ &\quad - \frac{1}{4} \partial_\mu G_{\alpha\beta} Y^\alpha Y^\beta (\eta + \bar{\eta}), \\ \delta Y^\mu &= -(\eta Y^\mu)' - \frac{1}{2} \Gamma_{\alpha\beta}^\mu Y^\alpha \bar{Y}^\beta (\eta + \bar{\eta}), \\ \delta \bar{Y}^\mu &= (\bar{\eta} \bar{Y}^\mu)' - \frac{1}{2} \Gamma_{\alpha\beta}^\mu Y^\alpha \bar{Y}^\beta (\eta + \bar{\eta}). \end{aligned} \quad (5)$$

Here $\Gamma_{\alpha\beta}^\mu$ are Christoffel symbols corresponding to the metric $G_{\mu\nu}$ and

$$Y^\mu = G^{\mu\nu} p_\nu - x'^\mu, \quad \bar{Y}^\mu = G^{\mu\nu} p_\nu + x'^\mu. \quad (6)$$

In terms of Y^μ, \bar{Y}^μ the constraints (2) have the form

$$L = \frac{1}{4} G_{\mu\nu} Y^\mu Y^\nu, \quad \bar{L} = \frac{1}{4} G_{\mu\nu} \bar{Y}^\mu \bar{Y}^\nu,$$

where $Y_\mu = G_{\mu\nu}Y^\nu$, $\bar{Y}_\mu = G_{\mu\nu}\bar{Y}^\nu$. The fields Y^μ and \bar{Y}^μ are the generalization of known Fubini fields very convenient for further analysis and will be.

3. Canonical quantization of the theory (1) was considered in ref.[18]. Operators of the constraints L and \bar{L} represent complicated functions of creation and annihilation operators for oscillating modes of the string and coordinate and momentum operators for zero modes. We will assume that the operators L and \bar{L} are expressed in some normal form (for example, Wick ordering for oscillating modes and Weyl ordering for zero ones [12, 18]). The operator Ω is given by the eq.(4) where L and \bar{L} are operators of the constraints and $\eta, \mathcal{P}, \bar{\eta}, \bar{\mathcal{P}}$ are ghost operators satisfying known commutation relations.

Within the BRST-BFV method the theory is anomalous if the following relations are fulfilled

$$[\Omega, \Omega] = A \neq 0, \quad (7)$$

$$[\Omega, H_T] = A_H \neq 0. \quad (8)$$

Here $[\ , \]$ is supercommutator, $H_T = H_0 + [\Psi, \Omega]$ is the total hamiltonian and Ψ is a gauge fermion [9-12]. If A and A_H are equal to zero the theory is called anomaly free. Operators A and A_H are called anomalies.

Obviously, the operator Ω must obey the operatorical (super)Jacobi identity regardless of whether the theory is anomalous or not:

$$[\Omega, [\Omega, \Omega]] = 0 \quad (9)$$

The eqs.(7,9) allow to write the equation for anomaly A :

$$[\Omega, A] = 0. \quad (10)$$

As for the eq.(8), in string theories A_H does not represent an independent anomaly because the canonical hamiltonian H_0 equals zero and so

$$H_T = [\Psi, \Omega]. \quad (11)$$

Therefore, the vanishing of A_H (8) is a consequence of the vanishing of A and the identity (9).

We will show that the eq.(10) is very powerful restriction on the form of anomaly A and allows to write down the A explicitly up to an arbitrary function having a definite tensor structure and depending only on x^μ . Using eq.(4,10) one can get the anomaly A as follows

$$\begin{aligned} A = \int d\sigma d\sigma' \Big\{ & \eta(\sigma)\eta(\sigma')([L(\sigma), L(\sigma')] + i\hbar(L(\sigma) + L(\sigma'))\delta'(\sigma - \sigma')) + \\ & + \bar{\eta}(\sigma)\bar{\eta}(\sigma')([\bar{L}(\sigma), \bar{L}(\sigma')] - i\hbar(\bar{L}(\sigma) + \bar{L}(\sigma'))\delta'(\sigma - \sigma')) + \\ & + i\hbar^2 \frac{13}{12\pi}(\eta(\sigma)\eta(\sigma') - \bar{\eta}(\sigma)\bar{\eta}(\sigma'))\delta'''(\sigma - \sigma') + \\ & + (\eta(\sigma)\bar{\eta}(\sigma') - \bar{\eta}(\sigma)\eta(\sigma'))[L(\sigma), \bar{L}(\sigma')] \Big\} \end{aligned} \quad (12)$$

We see that the form of anomaly is defined by the commutators $[L, L]$, $[\bar{L}, \bar{L}]$ and $[L, \bar{L}]$.

It is convenient to carry out further analysis in terms of symbols of operators. Let B and C are some operators depending on operators of canonical variables and written in some normal form. Any operator can be associated with a c -valued function of classical arguments $\Gamma^M \equiv (x^\mu(\sigma), p_\mu(\sigma))$ called the normal symbol of the operator (see for details [12,21]). Let $B(\Gamma)$ and $C(\Gamma)$ be symbols of the operators B and C . Symbol corresponding to the product BC is called $*$ -product of the symbols $B(\Gamma)$ and $C(\Gamma)$ and has the form

$$B(\Gamma) * C(\Gamma) = \exp \left\{ \Gamma_1^M \Gamma_2^N \frac{\delta}{\delta \Gamma_1^M} \frac{\delta}{\delta \Gamma_2^N} \right\} B(\Gamma_1) C(\Gamma_2) \Big|_{\Gamma_1 = \Gamma_2 = \Gamma} \quad (13)$$

Here $\Gamma'^M \equiv (x^\mu(\sigma'), p_\mu(\sigma'))$; the sum over M, N implies the summation over μ, ν and integration over σ, σ' . The contraction in (13) corresponds to the choosed type of ordering. The symbol of commutators $[B, C]$ is $B(\Gamma) * C(\Gamma') - C(\Gamma') * B(\Gamma)$ and is called $*$ -commutator.

Let us consider $*$ -commutator corresponding to the commutator $[L(\sigma), L(\sigma')]$. Taking into account eq.(13) one obtains

$$L(\sigma) * L(\sigma') - L(\sigma') * L(\sigma) = i\hbar \{L(\sigma), L(\sigma')\} + A^{(1)}(\sigma, \sigma'). \quad (14)$$

The function $A^{(1)}(\sigma, \sigma')$ appends when we expand the exponential in eq.(13) in series in power of contractions. Since every contractions contains \hbar , $*$ -commutator should be in general case an infinite series in powers of \hbar . The first term in (13) is proportional to classical Poisson bracket, the following terms correspond to loop corrections. In free string theory the constraints are quadratic function of Γ^M . It means that the series in \hbar terminetes of the second order and we get the known result for the free string anomaly. In the string theory interacting with arbitrary background fields the series for $A^{(1)}(\sigma, \sigma')$ contains all orders in \hbar and its straightforward calculation is difficult and complicated enough problem. Symbols corresponding to commutators $[\bar{L}(\sigma), \bar{L}(\sigma')]$ and $[L(\sigma), \bar{L}(\sigma')]$ have structure analogous to (14).

For further analysis it is useful to take into account the dimensions of the basic objects of the theory

$$\begin{aligned} \dim x^\mu &= 0, & \dim p_\mu &= 1, & \dim \eta &= \dim \bar{\eta} = c, \\ \dim \mathcal{P} &= \dim \bar{\mathcal{P}} = 1 - c. \end{aligned} \quad (15)$$

Then

$$\dim L = \dim \bar{L} = 2, \quad \dim \Omega = 1 + c, \quad \dim A = 2 + 2c. \quad (16)$$

Let the functions $A^{(2)}(\sigma, \sigma')$ and $A^{(3)}(\sigma, \sigma')$ are given by the equations

$$\begin{aligned} \bar{L}(\sigma) * \bar{L}(\sigma') - \bar{L}(\sigma') * \bar{L}(\sigma) &= i\hbar \{ \bar{L}(\sigma), \bar{L}(\sigma') \} + A^{(2)}(\sigma, \sigma'). \\ L(\sigma) * \bar{L}(\sigma') - \bar{L}(\sigma') * L(\sigma) &= A^{(3)}(\sigma, \sigma') \end{aligned} \quad (17)$$

and the functions $A^{(1)}(\sigma, \sigma')$ is given by the eq.(14). Let \mathcal{A} is a symbol corresponding to the operator of anomaly A . Taking into account the eq.(12) we obtain

$$\mathcal{A} = \int d\sigma d\sigma' \left\{ \eta(\sigma) \eta(\sigma') [A^{(1)}(\sigma, \sigma') + i\hbar^2 \frac{13}{12\pi} \delta'''(\sigma - \sigma')] + \right.$$

$$\begin{aligned}
& + \bar{\eta}(\sigma)\bar{\eta}(\sigma') [A^{(2)}(\sigma, \sigma') + i\hbar^2 \frac{13}{12\pi} \delta'''(\sigma - \sigma')] + \\
& + [\eta(\sigma)\bar{\eta}(\sigma') - \bar{\eta}(\sigma)\eta(\sigma')] A^{(3)}(\sigma, \sigma') \Big\}.
\end{aligned} \tag{18}$$

Here $\dim A^1 = \dim A^2 = \dim A^3 = 4$ and $\eta(\sigma), \bar{\eta}(\sigma')$ are classical anticommuting functions. The eqs.(14,17) show that $A^{(1)}(\sigma, \sigma') = -A^{(1)}(\sigma', \sigma)$, $A^{(2)}(\sigma, \sigma') = -A^{(2)}(\sigma', \sigma)$. It allows to write

$$\begin{aligned}
A^{(1)}(\sigma, \sigma') &= (f_1(\sigma) + f_1(\sigma')\delta'(\sigma - \sigma') + (f_3(\sigma) + f_3(\sigma')\delta'''(\sigma - \sigma')), \\
A^{(2)}(\sigma, \sigma') &= (g_1(\sigma) + g_1(\sigma')\delta'(\sigma - \sigma') + (g_3(\sigma) + g_3(\sigma')\delta'''(\sigma - \sigma')),
\end{aligned} \tag{19}$$

where arbitrary functions f_1, g_1, f_3, g_3 have dimensions

$$\dim f_1 = \dim g_1 = 2, \quad \dim f_3 = \dim g_3 = 0.$$

As to the function $A^{(3)}(\sigma, \sigma')$ it has more general structure

$$\begin{aligned}
A^{(3)}(\sigma, \sigma') &= \frac{1}{2}(h_0(\sigma) + h_0(\sigma'))\delta(\sigma - \sigma') + \frac{1}{2}(h_1(\sigma) + h_1(\sigma'))\delta'(\sigma - \sigma') + \\
&+ \frac{1}{2}(h_2(\sigma) + h_2(\sigma'))\delta''(\sigma - \sigma') + \frac{1}{2}(h_3(\sigma) + h_3(\sigma'))\delta'''(\sigma - \sigma'),
\end{aligned} \tag{20}$$

where arbitrary functions h_k ($k = 0, 1, 2, 3$) have dimensions $\dim h_k = 3 - k$.

Using the eqs.(19,20) we can write the anomaly \mathcal{A} as follows

$$\begin{aligned}
\mathcal{A} &= \int d\sigma d\sigma' \left\{ i\hbar^2 \frac{13}{12\pi} (\eta\eta''' + \bar{\eta}\bar{\eta}''') + a_0 + a_1 + a_2 + a_3 \right\}, \\
a_0 &= 2\eta\bar{\eta} h_0, \\
a_1 &= \eta\eta' f_1 + \bar{\eta}\bar{\eta}' g_1 + 2(\eta\bar{\eta}' - \bar{\eta}\eta') h_1, \\
a_2 &= 2(\eta\bar{\eta}'' - \bar{\eta}\eta'') h_2, \\
a_3 &= \eta\eta''' f_3 + \bar{\eta}\bar{\eta}''' g_3 + 2(\eta\bar{\eta}''' - \bar{\eta}\eta''') h_3.
\end{aligned} \tag{21}$$

Thus we obtained the anomaly up to unknown functions $h_0, f_1, h_1, g_1, h_2, f_3, h_3, g_3$.

However we have not yet used the eq.(10). Being rewritten in terms of symbols of operators the eq.(10) leads to equation of the following type

$$\hat{\delta}\mathcal{A} = 0, \tag{22}$$

where the operator $\hat{\delta}$ has the structure

$$\hat{\delta} = \sum_{n=0}^{\infty} \hbar^n \delta_n. \tag{23}$$

Here $\delta_0 = \delta$ is the operator of classical BRST-transformations (5). An explicit form of δ_n at $n > 0$ is unessential for our purposes.

The functional \mathcal{A} can be represented as follows

$$\mathcal{A} = \sum_{n=1}^{\infty} \hbar^{n+1} \mathcal{A}_n, \tag{24}$$

where \mathcal{A}_n are quantum correction to the anomaly. It is easy to show that the eq.(22-24) lead to

$$\delta\mathcal{A}_1 = 0 \quad (25)$$

$$\delta\mathcal{A}_n = G_n, \quad n \geq 2 \quad (26)$$

$$G_n = \sum_{m=1}^n \delta_m \mathcal{A}_{n-m}.$$

The eq.(25) is one for the first (one-loop) quantum correction, the eq.(26) allows to find higher quantum corrections to the anomaly.

4. Our aim is to investigate the structure of the eqs.(25,26) solutions. We start with eq.(25) defining the one-loop contribution to the anomaly.

Let \mathcal{A}_1 is a solution of the eq.(25). According to the previous analysis general structure of \mathcal{A}_1 is given by the eq.(21). It means that to satisfy the eq.(25) we should demand

$$\delta a = \partial_\sigma \chi \quad (27)$$

with some function χ , $\dim \chi = \dim a - 1$. The conditions (27) impose strict constraints on the functions $h_0, f_1, h_1, g_1, h_2, f_3, h_3, g_3$.

Let $\Psi(x, p)$ is any of the above functions with the dimension $\dim \Psi$. Since the only dimensional quantities are x'^μ, p_μ and ∂_σ we can write the Ψ as a polynomial of a power $\dim \Psi$ in $Y_\mu, \bar{Y}_\mu, Y'_\mu$ and \bar{Y}'_μ . For example

$$f_1 = \beta_{\mu\nu} Y^\mu \bar{Y}^\nu + \alpha_{\mu\nu} Y^\mu Y^\nu + \bar{\alpha}_{\mu\nu} \bar{Y}^\mu \bar{Y}^\nu + \gamma_\mu Y'^\mu + \bar{\gamma}_\mu \bar{Y}'^\mu, \quad (28)$$

where $\beta_{\mu\nu}, \alpha_{\mu\nu}, \bar{\alpha}_{\mu\nu}, \gamma_\mu, \bar{\gamma}_\mu$ are arbitrary functions of x . Analogous relations can be written for $h_0, f_1, h_1, g_1, h_2, f_3, h_3, g_3$. In particular, the functions f_3, g_3, h_3 are independent on Y, \bar{Y} and their derivatives at all. Taking into account the eqs.(5) we obtain

$$\delta\Psi = \sum_{n=0}^{\dim \Psi} (\Psi^{(n)} \partial_\sigma^n \eta + \bar{\Psi}^{(n)} \partial_\sigma^n \bar{\eta}) \quad (29)$$

with some functions $\Psi^{(n)}, \bar{\Psi}^{(n)}$.

Let us consider the eqs.(27). It is obviously that the function χ should have the structure

$$\chi = \sum (ghost) \lambda(x, p), \quad (30)$$

where $(ghost)$ is a suitable ghost contribution and λ are polynoms of the proper power in Y, \bar{Y}, Y', \bar{Y}' with coefficients depending on x . Namely, the power equals $\dim a - \dim(ghost) - 1$. Taking into account the eq.(27, 30) one gets

$$\sum \delta a = \sum [(ghost)' \lambda + (ghost) \lambda']. \quad (31)$$

δa can be found on the base of the eqs.(5, 21, 29). It is obvious that δa will have the special structure given by the right hand side of the eq.(31) only under special restrictions

on the functions Ψ . A straightforward but complicated enough consideration leads to the following result

$$\begin{aligned} a_0 &= a_2 = 0, \\ a_1 &= (\eta + \bar{\eta})(\eta - \bar{\eta})' \beta_{\mu\nu} Y^\mu \bar{Y}^\nu, \\ a_3 &= (\eta \eta''' \Lambda + \bar{\eta} \bar{\eta}''' \bar{\Lambda}). \end{aligned} \quad (32)$$

Here $\beta_{\mu\nu}$ is an arbitrary function of x and $\Lambda, \bar{\Lambda}$ are the constants. The corresponding χ is

$$\chi = i\bar{\eta}\eta (\eta - \bar{\eta})' \beta_{\mu\nu} Y^\mu \bar{Y}^\nu \epsilon + i(\bar{\eta}\eta' \bar{\eta}'' \bar{\Lambda} - \eta\eta' \eta'' \Lambda) \epsilon. \quad (33)$$

Thus the one-loop correction to the anomaly has the form

$$\begin{aligned} \mathcal{A}_1 = i\hbar^2 \int d\sigma \left\{ (\eta + \bar{\eta})(\eta - \bar{\eta})' \beta_{\mu\nu}^{(1)}(x) Y^\mu \bar{Y}^\nu + \right. \\ \left. + \eta\eta''' \beta^{(1)} + \bar{\eta}\bar{\eta}''' \bar{\beta}^{(1)} \right\}. \end{aligned} \quad (34)$$

where $\beta^{(1)} = \frac{13}{12\pi} + \Lambda^{(1)}, \bar{\beta}^{(1)} = \frac{13}{12\pi} + \bar{\Lambda}^{(1)}$ and the index (1) indicates the one-loop contribution. The eq.(34) is general consequence of the BRST-BFV quantization procedure.

5. Let us consider now the eqs.(26) defining the higher quantum corrections. Writing

$$\mathcal{A}_n = \int d\sigma a^{(n)}, \quad G_n = \int d\sigma g^{(n)}, \quad (35)$$

we obtain using the eq.(26) $\delta a^{(n)} = g^{(n)} + \partial_\sigma \lambda^{(n)}$ where $\lambda^{(n)}$ are some functions. Since $\delta^2 a^{(n)} = 0$ one gets $\delta g^{(n)} = -\partial_\sigma \delta \lambda^{(n)}$. A solution of this equation can be written as

$$g^{(n)} = g_0^{(n)} + \partial_\sigma \kappa^{(n)}, \quad (36)$$

where $\kappa^{(n)}$ are some functions and

$$\delta g_0^{(n)} = 0. \quad (37)$$

The eqs.(35,36) show that

$$G_n = \int d\sigma g_0^{(n)}. \quad (38)$$

A solution $g_0^{(n)}$ of the eq.(37) can be found taking into account that a ghost structure of $g_0^{(n)}$ is defined by the ghost structure $\delta a^{(n)}$ and the dimension of $g_0^{(n)}$ equals $\dim g_0^{(n)} = 4 + 3c$. The eqs.(18 - 21) allow to write $a^{(n)} = a_0 + a_1 + a_2 + a_3$ where a_k are given by (21). Calculating $\delta a^{(n)}$ we find the general form of $g_0^{(n)}$

$$\begin{aligned} g_0^{(n)} &= \bar{\eta}\eta\eta' B + \bar{\eta}\eta\bar{\eta}' \bar{B} + \bar{\eta}\eta\eta'' C + \bar{\eta}\eta\bar{\eta}'' \bar{C} + \bar{\eta}\eta\eta''' D + \bar{\eta}\eta\bar{\eta}''' \bar{D} + \\ &+ \bar{\eta}\eta\eta^{(4)} E + \bar{\eta}\eta\bar{\eta}^{(4)} \bar{E} + \eta\eta'\eta' F + \bar{\eta}\bar{\eta}'\eta' \bar{F} + \eta\eta'\eta'' H + \bar{\eta}\bar{\eta}'\eta'' \bar{H} + \\ &+ \eta\eta'\eta''' S + \bar{\eta}\bar{\eta}'\eta''' \bar{S} + \eta\bar{\eta}'\eta'' T + \bar{\eta}\eta'\eta'' \bar{T} + \eta\bar{\eta}'\eta'' R + \bar{\eta}\eta'\eta'' \bar{R} + \\ &+ \eta\bar{\eta}'\eta''' U + \bar{\eta}\eta'\eta''' \bar{U} + \eta\eta'\eta'' V \bar{\eta}\bar{\eta}'\eta'' \bar{V} + \eta\eta'\eta''' W + \bar{\eta}\bar{\eta}'\eta''' \bar{W} \end{aligned} \quad (39)$$

where $\Phi \equiv (B, \bar{B}, C, \bar{C}, D, \bar{D}, E, \bar{E}, F, \bar{F}, H, \bar{H}, S, \bar{S}, T, \bar{T}, R, \bar{R}, U, \bar{U}, V, \bar{V}, W, \bar{W})$ are some unknown functions of x, p of definite dimensions. The functions Φ can be presented as polynomials in power of monomials $Y^\mu, \bar{Y}^\mu, Y'^\mu, \bar{Y}'^\mu, Y''^\mu, \bar{Y}''^\mu, Y'''^\mu, \bar{Y}'''^\mu$. The number of monomials is defined by dimension of Φ with unknown coefficients depending only on x^μ . The eq.(39) and the structure of the functions Ψ allow to calculate $\delta g_0^{(n)}$ in explicit form. Then the eq.(3) will lead to a system of the linear homogenous equations for the functions Φ . The only solution of this system is $\Phi = 0$. It means that $g_0^{(n)} = 0$ and, hence, $G_n = 0$ too.

As a result of above discussion the eqs.(26) defining higher quantum correction to the anomaly are reduced to

$$\delta \mathcal{A}_n = 0. \quad (40)$$

But the equation of this type has been studied in previous section. Therefore we are able to write down the most general form of the anomaly

$$\mathcal{A} = i\hbar^2 \int d\sigma \left\{ (\eta + \bar{\eta})(\eta - \bar{\eta})' \beta_{\mu\nu}(x) Y^\mu \bar{Y}^\nu + \eta \eta''' \beta + \bar{\eta} \bar{\eta}''' \bar{\beta} \right\}, \quad (41)$$

where

$$\begin{aligned} \beta_{\mu\nu}(x) &= \sum_{n=0}^{\infty} \hbar^n \beta_{\mu\nu}^{(n)}(x), \\ \beta &= \sum_{n=0}^{\infty} \hbar^n \beta^{(n)}, \quad \bar{\beta} = \sum_{n=0}^{\infty} \hbar^n \bar{\beta}^{(n)}. \end{aligned} \quad (42)$$

Here $\beta_{\mu\nu}^{(n)}(x)$ are arbitrary functions and $\beta^{(n)}, \bar{\beta}^{(n)}$ are the arbitrary constants. Thus we see that the general form of anomaly (41) can be established up to above functions and constants using only the generic capabilities of generalized canonical quantization. Certainly, to find these functions and constants we should perform some kind of loop calculation. The example of such calculations was given in ref.[18]. It is evident that the found general form of anomaly (41) in any case allows to simplify the process of real loop calculations.

6. Now we are going to investigate an arbitrariness in the anomaly structure (41). Let \mathcal{A} be a solution of the equation $\delta \mathcal{A} = 0$. Then $\tilde{\mathcal{A}} = \mathcal{A} + \delta \Theta$ with any Θ is also a solution of this equation. Writing

$$\tilde{\mathcal{A}} = i\hbar^2 \int d\sigma \tilde{a}, \quad \mathcal{A} = i\hbar^2 \int d\sigma a, \quad \Theta = i\hbar^2 \int d\sigma \theta,$$

we obtain

$$\tilde{a} = a + \delta \theta + \partial_\sigma \rho$$

with some ρ . Here $\dim \theta = 2 + c, \dim \rho = 2 + 2c$. Hence

$$\begin{aligned} &(\eta + \bar{\eta})(\eta - \bar{\eta})' \tilde{\beta}_{\mu\nu}(x) Y^\mu \bar{Y}^\nu + \eta \eta''' \tilde{\beta} + \bar{\eta} \bar{\eta}''' \tilde{\bar{\beta}} = \\ &= (\eta + \bar{\eta})(\eta - \bar{\eta})' \beta_{\mu\nu}(x) Y^\mu \bar{Y}^\nu + \eta \eta''' \beta + \bar{\eta} \bar{\eta}''' \bar{\beta} + \delta \theta + \partial_\sigma \rho. \end{aligned} \quad (43)$$

The dimensions of θ and ρ allow to write

$$\begin{aligned}\theta &= \eta M + \bar{\eta} \bar{M} + \eta' N + \bar{\eta}' \bar{N} + \eta'' P + \bar{\eta}'' \bar{P} \\ \rho &= \eta \eta' K + \bar{\eta} \bar{\eta}' \bar{K} + \eta \bar{\eta} L + \eta' \bar{\eta} Q + \bar{\eta}' \eta' \bar{Q} + \\ &\quad + \eta \eta'' V + \bar{\eta} \bar{\eta}'' \bar{V} + \eta'' \bar{\eta} Z + \bar{\eta}'' \eta' \bar{Z},\end{aligned}\tag{44}$$

where $\Phi \equiv (M, \bar{M}, N, \bar{N}, P, \bar{P})$ and $\chi \equiv (K, \bar{K}, L, Q, \bar{Q}, V, \bar{V}, Z, \bar{Z})$ are some functions with definite dimensions.

A straightforward calculation shows that $\partial_\sigma \rho$ have the ghost structure given by the right hand side of the eq.(43) only if $\chi = 0$ and hence $\rho = 0$. Now let us consider the structure of $\delta\theta$. According to the eq.(44) one gets $\dim M = \dim \bar{M} = 2, \dim N = \dim \bar{N} = 1$ and $\dim P = \dim \bar{P} = 0$. Therefore M, \bar{M} can be expanded in the basis of $Y^\mu \bar{Y}^\nu, Y^\mu Y^\nu, \bar{Y}^\mu \bar{Y}^\nu, Y'^\mu, \bar{Y}'^\mu$; the N, \bar{N} can be expanded in the basis of Y^μ, \bar{Y}^μ and P, \bar{P} are arbitrary functions of x . Taking into account the properties of Φ , calculating $\delta\theta$ and demanding that $\delta\theta$ has the ghost structure given by the right hand side of eq.(43) one gets

$$\begin{aligned}\theta &= \eta \left(-\frac{1}{18} \partial_\mu \xi_\nu Y^\mu \bar{Y}^\nu + \frac{1}{18} \partial_\mu \xi_\nu Y^\mu Y^\nu - \frac{1}{9} \xi_\mu Y'^\mu \right) + \\ &\quad + \bar{\eta} \left(-\frac{1}{18} \partial_\mu \xi_\nu Y^\mu \bar{Y}^\nu + \frac{1}{18} \partial_\mu \xi_\nu \bar{Y}^\mu \bar{Y}^\nu + \frac{1}{9} \bar{\xi}_\mu \bar{Y}'^\mu \right) - \\ &\quad - \frac{1}{3} \eta' \xi_\mu Y^\mu + \frac{1}{3} \bar{\eta}' \xi_\mu \bar{Y}^\mu,\end{aligned}\tag{45}$$

where $\xi_\mu(x)$ is an arbitrary vector field satisfying the relation $\nabla_\mu \xi_\nu(x) = \nabla_\nu \xi_\mu(x)$. Here $\nabla_\mu \xi_\nu(x)$ is a covariant derivative in terms of background metric $G_{\mu\nu}(x)$.

The eqs.(43, 45) together with $\rho = 0$ lead to

$$\begin{aligned}\tilde{\beta}_{\mu\nu}(x) &= \beta_{\mu\nu}(x) + \nabla_\mu \xi_\nu(x), \\ \tilde{\beta} &= \beta, \quad \tilde{\bar{\beta}} = \bar{\beta}.\end{aligned}\tag{46}$$

The eqs.(46) define the admissible ambiguity in general structure of the anomaly (41).

7. We have considered the problem of anomaly in string theory interacting with background gravitational field in framework of generalized canonical quantization procedure. This problem has been formulated in terms of symbols of operators as a problem of solution of the equation $\delta \mathcal{A}(\eta, \bar{\eta}, x, p) = 0$ where δ is the canonical BRST - transformation (5) and \mathcal{A} is a symbol of the anomaly operator. A general solution of above equation has been obtained and an admissible arbitrariness in the solution has been described. We see, in principle, that the anomaly problem is reduced to a cohomology problem for the operator δ (5) acting in space of c-valued functions depending on coordinates of the extended phase space.

Taking into account the general structure of the anomaly (41) one can immediately write the conditions of anomaly cancellation in the form $\beta_{\mu\nu}(x) = 0, \beta = 0, \bar{\beta} = 0$. As we have already mentioned the functions $\beta_{\mu\nu}(x)$ and the constants $\beta, \bar{\beta}$ are not defined within the approach under consideration. Therefore, we should develop a calculational technique allowing to compute the function $\beta_{\mu\nu}$ and the constants $\beta, \bar{\beta}$ only on the base

of the generalized canonical quantization scheme. The first step towards such a technique was undertaken in ref.[18].

The approach general and allows to investigate various string models in background fields. In particular, inclusion of massless antisymmetric tensor field in to the theory will not lead to any problems and will not demand essential modification of the formalism.

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